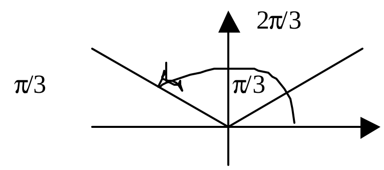
7.6 SOLVING TRIG EQUATIONS

Algebraic methods and trig identities are used to find solutions to trig equations. Methods used include factoring, quadratic formula and squaring.

Example.

1.) Solve $2\sin x = \sqrt{3}$, where $0 \leq x < 2\pi$.



$2 \sin x = \sqrt{3} \longrightarrow \sin x = \sqrt{3} / 2$

$\longrightarrow x = \pi / 3,\ 2\pi / 3.$

2.) Solve $2 \sin^2 x + 1 = 3 \sin x$, where $0 \leq x < 2\pi$.

Let $U = \sin x$ then

$LHS = 2U^2 - 3U + 1 = 0.$

Using the quadratic formula get

$$U = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9 - 8}}{4}$$

$$= \frac{3 \pm 1}{4} = \tfrac{1}{2},\ 1$$

But $U = \sin x$ so that

$\sin x = \tfrac{1}{2} \longrightarrow x = \pi / 6,\ 5\pi / 6$

$\sin x = 1 \longrightarrow x = \pi / 2,\ 3\pi / 2.$

3.) Solve $2 \sin x - \cos x = 1$ where $0 \leq x < 360°$, round angles to the nearest tenth.

(#51 p471)

Square both sides of the equation.

$(2 \sin x - \cos x)^2 = 1$

$4 \sin^2 x - 4 \sin x \cos x + \cos^2 x = 1$

$3 \sin^2 x + \sin^2 x + \cos^2 x - 4\sin x \cos x = 1$

$3 \sin^2 x + 1 - 4 \sin x \cos x = 1$

$3 \sin^2 x - 4 \sin x \cos x = 0$

$\sin x (3 \sin x - 4\cos x) = 0$

Set each factor to zero.

1.) $\sin x = 0$ → $x = 0\ ,\ 180°$

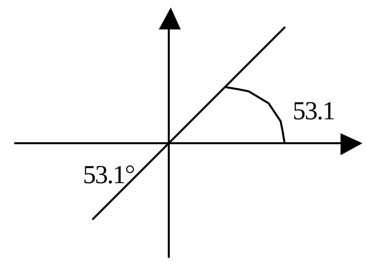
2.) $3 \sin x - 4 \cos x = 0$

$3 \sin x = 4 \cos x$ → $\sin x / \cos x = 4/3$

$\tan x = 4/3$

tan x is positive in quadrants I and III. Therefore
x is in QI and Q III. Use $\tan^{-1} 4/3$ on your calculator to get $x = 53.1°$.
Use the reference angle in QIII to get $x = 233.1°$.

$180° + 53.1° = 233.1°$



HW: p. 470ff. # 2-11(odds), 14, 23, 31, 41, 46, 91.