Hypothesis Testing

Hypothesis testing is a process of making decision or inference *from a set of sample data* ***for evaluating claims about a population***. For example, you try to investigate whether an individual's blood pressure is higher under stress. From your experiments, someone gets higher blood pressure after running or riding an exercise bike. But others may not. How do you use your data to support your claim? You must go through **statistical hypothesis testing** to make a conclusion about *the truth of your claim*.

Null Hypothesis:

The Null Hypothesis is denoted by H_0 and is assumed to be true until **sufficient evidence** is obtained **to warrant its rejection**. For example, there will be no increase on blood pressure after an exercise.

Alternative Hypothesis:

The Alternative Hypothesis is denoted by H_1 or H_a and is **opposite** of Null Hypothesis. It will be accepted as true if we can disprove H_0. The researcher usually wants to prove it. For example, there will be increase on blood pressure after an exercise.

The test statistic and its *p*-value:

- ***A single statistic*** calculated from the sample which will allow us to reject or not reject H_0, and
- ***A probability***, calculated from the test statistic that measures whether the test statistic is **likely** or **unlikely**, assuming H_0 is true.

The rejection region:

–A rule that tells us for which values of the test statistic, or for which *p*-values, the null hypothesis should be rejected.

Conclusion:

–Either "Reject H_0" or "Do not reject H_0", along with a statement about the reliability of your conclusion.

How do you decide when to reject H_0?

–Depends on the **significance level, α,** the maximum tolerable risk you want to have of making a mistake, if you decide to reject H_0.

–Usually, the significance level is $\alpha = .01$ or $\alpha = .05$.

Making Decision:

p-value $\le \alpha$, then **reject H_0**

p-value $> \alpha$, then **fail to reject H_0**

where α is the specified significance level.

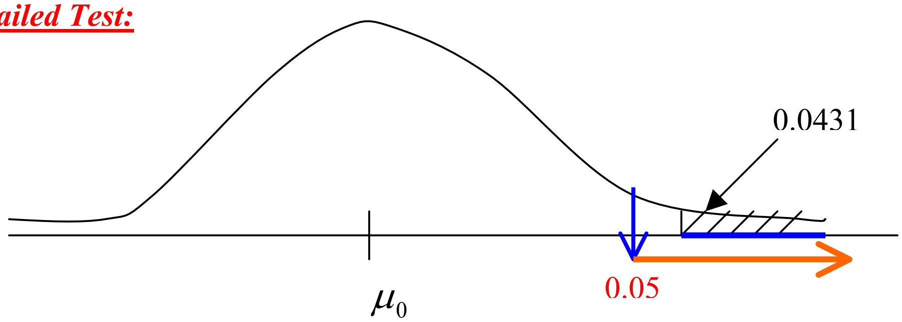
e.g.

If p-value = 0.0431, $\alpha = 0.05$, then reject H_0 at 5% significance level.

That is, under null hypothesis, there is 4.31% chance of getting the test statistic value. Since we allow **5% error margin** (i.e. significance level), we reject the null hypothesis based upon the test statistic value lied far away from μ_0.

The probability of obtaining data in the experiment is 0.0431under the null hypothesis is true.

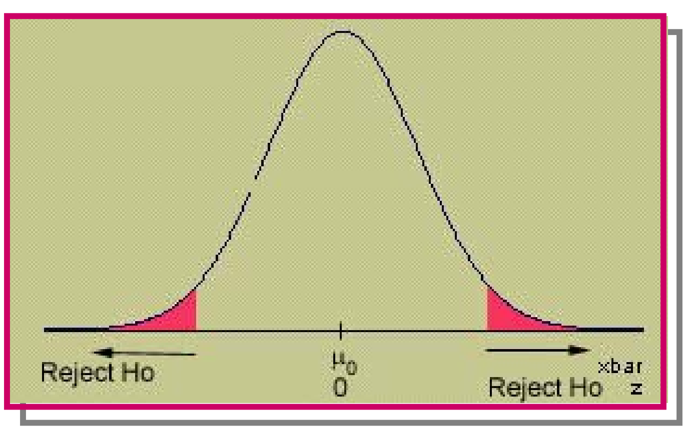
One Tailed Test:



Test Statistic for two tailed test:

If H_0 is true the value of $\bar{x}$ should be close to μ_0, and z will be close to 0.

If H_0 is false, $\bar{x}$ will be much larger or smaller than μ_0, and z will be much larger or smaller than 0, indicating that we should reject H_0.



Large-Sample Test Statistics Using the *z* Distribution

If $n \geq 30$, we use the test statistic $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ to test the parameter μ.

If $n \geq 30$, we use the test statistic $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ to test the parameter $\mu_1 - \mu_2$

Example

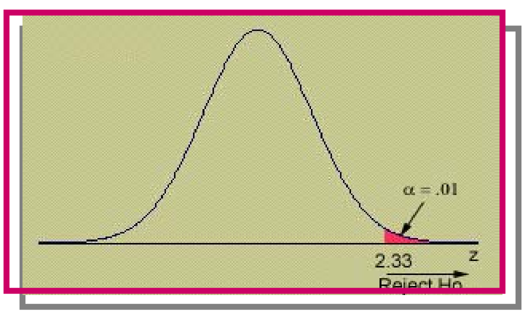
A homeowner randomly samples 64 homes similar to her own and finds that the average selling price is $252,000 with a standard deviation of $15,000. Is this sufficient evidence to conclude that the average selling price is greater than $250,000? Use α = .01.

$H_0 : \mu = 250{,}000$ vs. $H_a : \mu > 250{,}000$

Test Statistic: $z \approx \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{252{,}000 - 250{,}000}{15{,}000/\sqrt{64}} = 1.07$

Critical Value Approach

What is the critical value of *z* that cuts off exactly α= .01 in the right-tail of the *z* distribution?

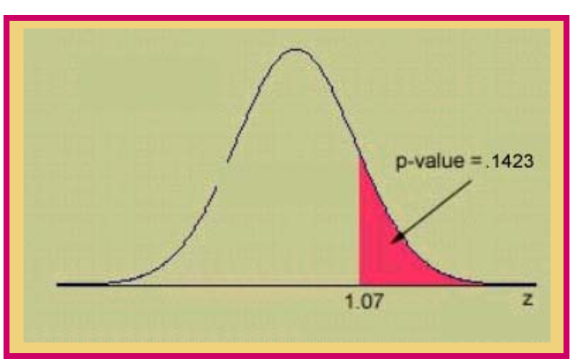


For our example, ***z* = 1.07 does not fall in the rejection region** and H_0 is not rejected. **There is not enough evidence to indicate that μ is greater than $250,000.**

Rejection Region: Reject H_0 if $z > 2.33$. If the test statistic falls in the rejection region, its *p*-value will be less than α = .01.

p-Value Approach

- The probability that our sample results or something even more unlikely would have occurred *just by chance*, when μ = 250,000.
  p-value: $P(z > 1.07) = 1 - .8577 = .1423$



Since the ***p*-value is greater than α = .01**, H_0 is not rejected. There is insufficient evidence to indicate that μ is greater than $250,000.

Example:

Average Daily Intakes	Men	Women
Sample size	50	50
Sample mean	756	762
Sample Std. Dev.	35	30

•Is there a difference in the average daily intakes of dairy products for men versus women? Use $\alpha = .05$.

$$H_0 : \mu_1 - \mu_2 = 0 \quad \text{(same)}$$
$$H_1 : \mu_1 - \mu_2 \neq 0 \quad \text{(different)}$$

$$\text{Test statistic: } z \approx \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{756 - 762 - 0}{\sqrt{\frac{35^2}{50} + \frac{30^2}{50}}} = -.92$$

p-Value Approach

- The probability of observing values of z that as far away from $z = 0$ as we have, *just by chance,* if indeed $\mu_1 - \mu_2 = 0$.

$p\text{-value: } P(z > .92) + P(z < -.92) = 2(.1788) = .3576$

Since the *p*-value is greater than $\alpha = .05$, H_0 is not rejected. There is insufficient evidence to indicate that men and women have different average daily intakes.